

# VIBE PROJECT

## Virtual Biomedical and STEM/STEAM Education

2021-1-HU01-KA220-HED-000032251



Funded by  
the European Union



Erasmus+

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PÉCSI TUDOMÁNYEGYETEM  
UNIVERSITY OF PÉCS

U.PORTO



Politechnika  
Śląska





# PATTERN RECOGNITION IN BIOMEDICAL ENGINEERING

MATHEMATICAL FOUNDATIONS



# Introduction





# Books



# The Term “Pattern Recognition”



Pattern Recognition



is a field whose objective is to assign an object or event to one of a few categories, based on features derived to emphasize commonalities. In practice, features are often extracted from sensory signals, such as images or audio.



is the act of taking in raw data and taking an action based on the category of the pattern.



# Terminology



What is the difference between

Image Processing,  
Image Recognition, and  
Pattern Recognition?



# Pattern recognition applications



Computer Vision



Character Recognition



Computer-aided Diagnosis



Speech Recognition



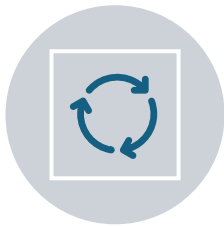
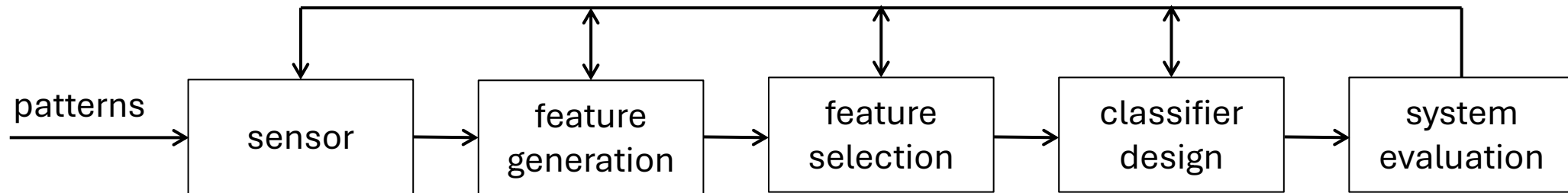
Data Mining and Knowledge Discovery



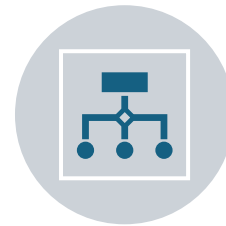
...



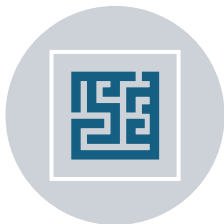
# Basic Stages of Pattern Analysis



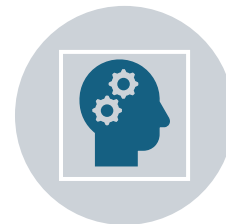
The stages are highly dependent on each other.



In order to design an optimal pattern recognition system, they all have to be optimised at once.



Patterns are analysed at different levels of abstraction.

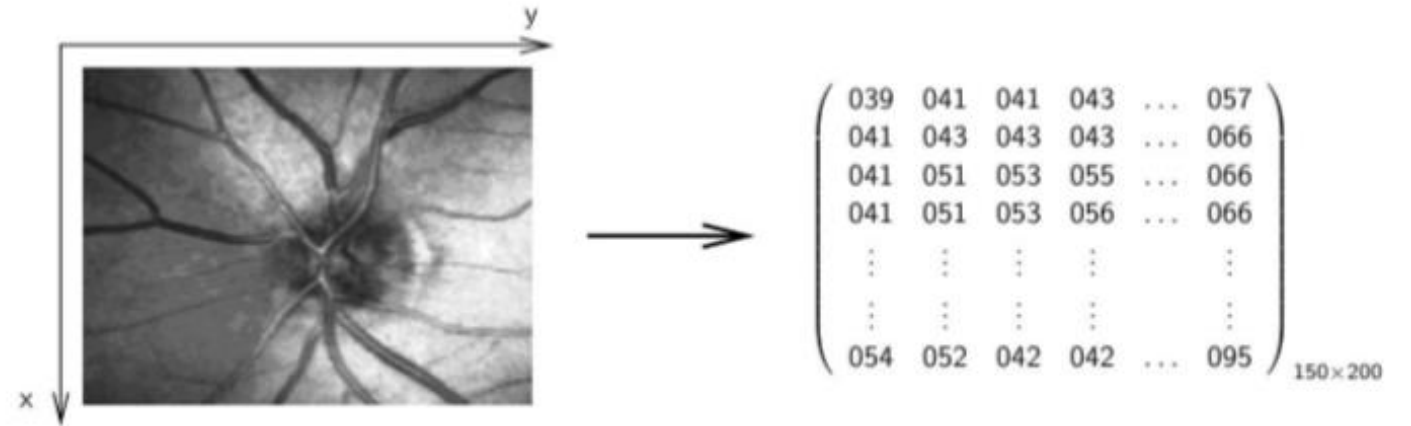


Integration of background knowledge into the process may be very useful.





# Low-Level Interpretation of Patterns

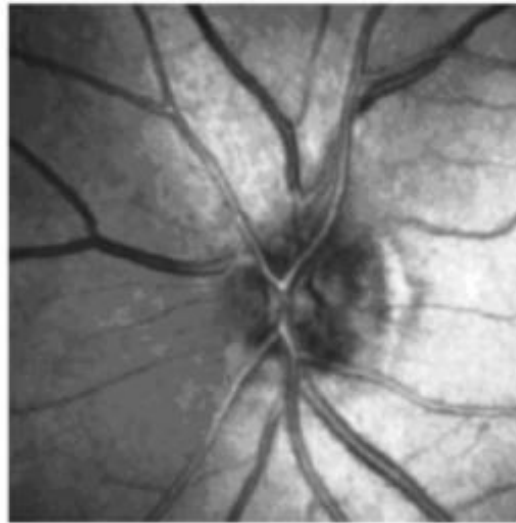


$$f(x, y) = \begin{pmatrix} f(0, 0) & f(0, 1) & \dots & f(0, 199) \\ f(1, 0) & f(1, 1) & \dots & f(1, 199) \\ \vdots & \vdots & \ddots & \vdots \\ f(149, 0) & f(149, 1) & \dots & f(149, 199) \end{pmatrix} ; \quad f(x, y) \in \{0, 1, 2, \dots, 255\}$$



# High-Level Interpretation of Patterns

Input Image



Gray Level Retina Image  
Papilla Shape - OK  
Blood Vessel Width - OK



# Basic Stages of Pattern Analysis



Optimization of the Entire  
Processing Chain at Once



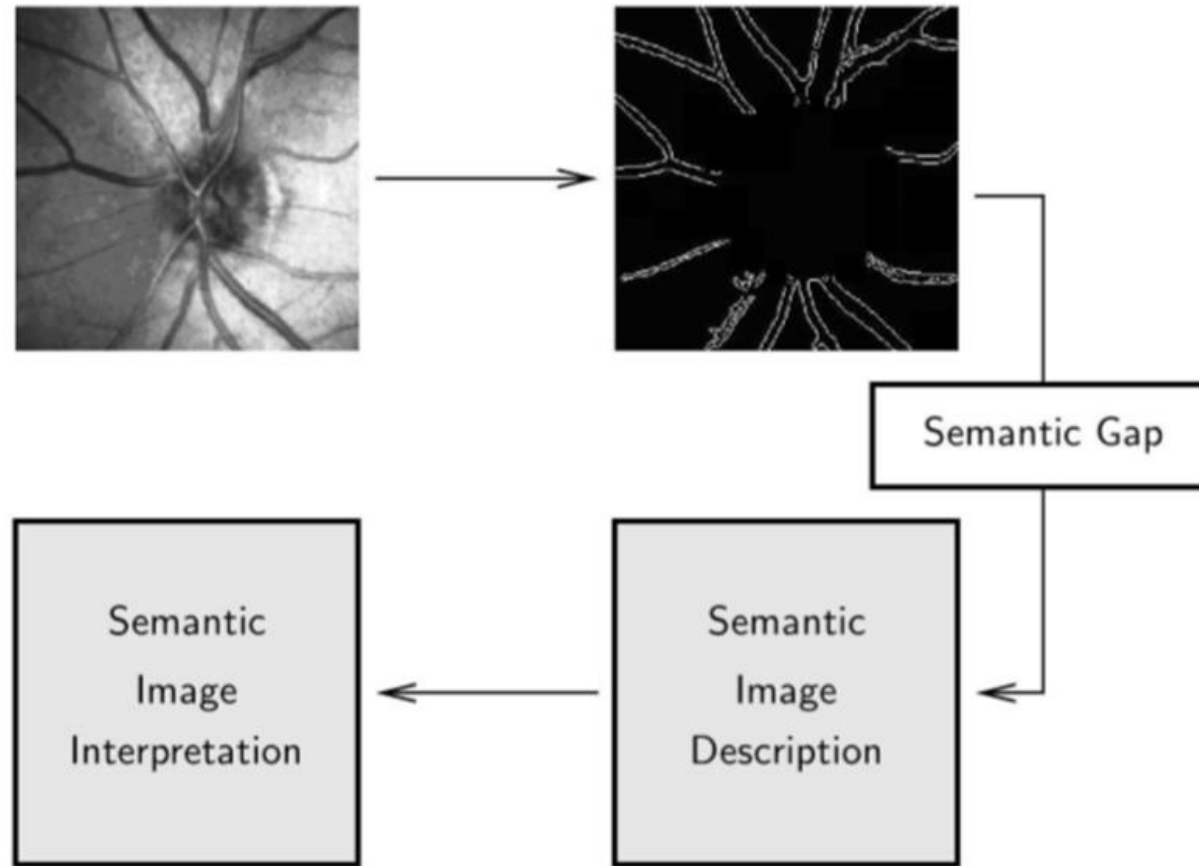
Combination of the Different  
Levels of Abstraction



Integration of Background  
Knowledge into the Process



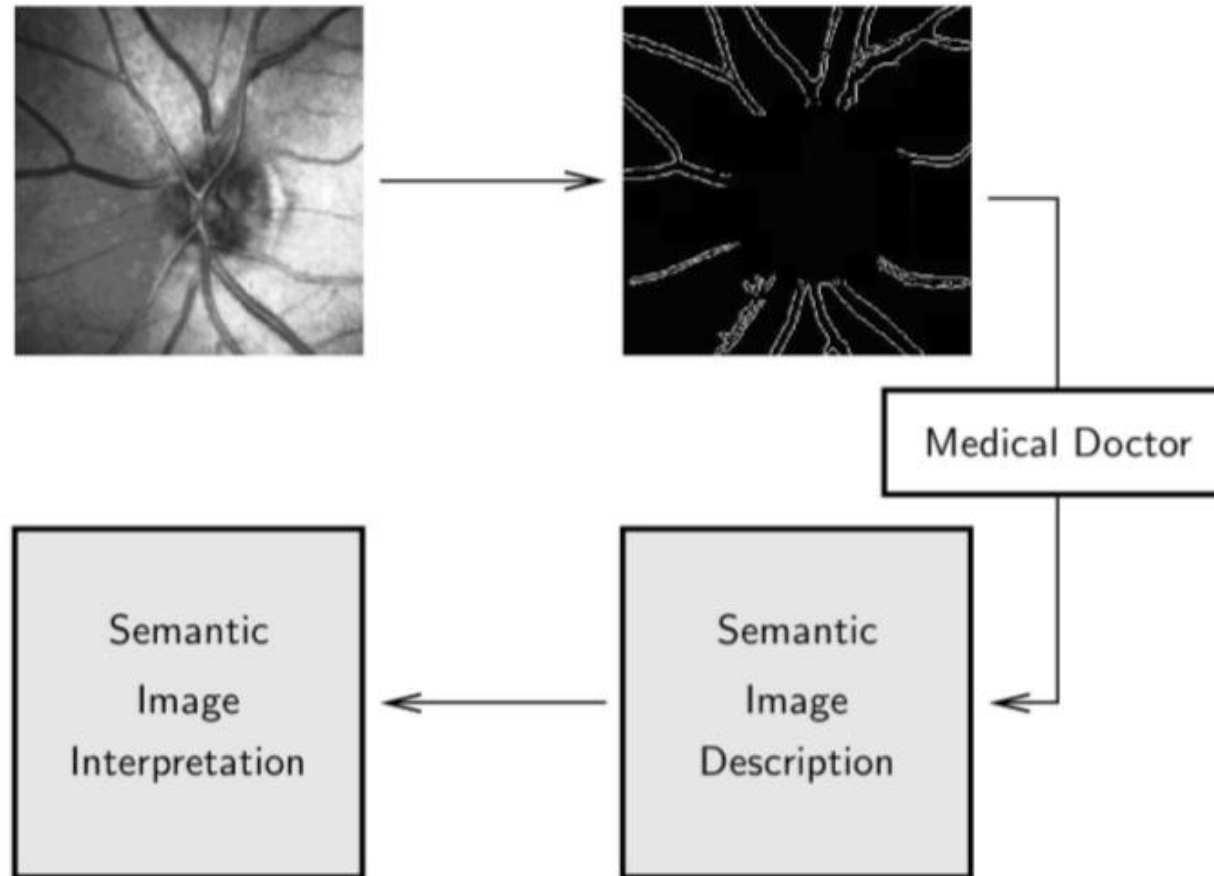
# Semantic Gap in Image Understanding



Grzegorz Marcin & Doniec Rafał, (2024). *Pattern Recognition*. University: Universität zu Lübeck



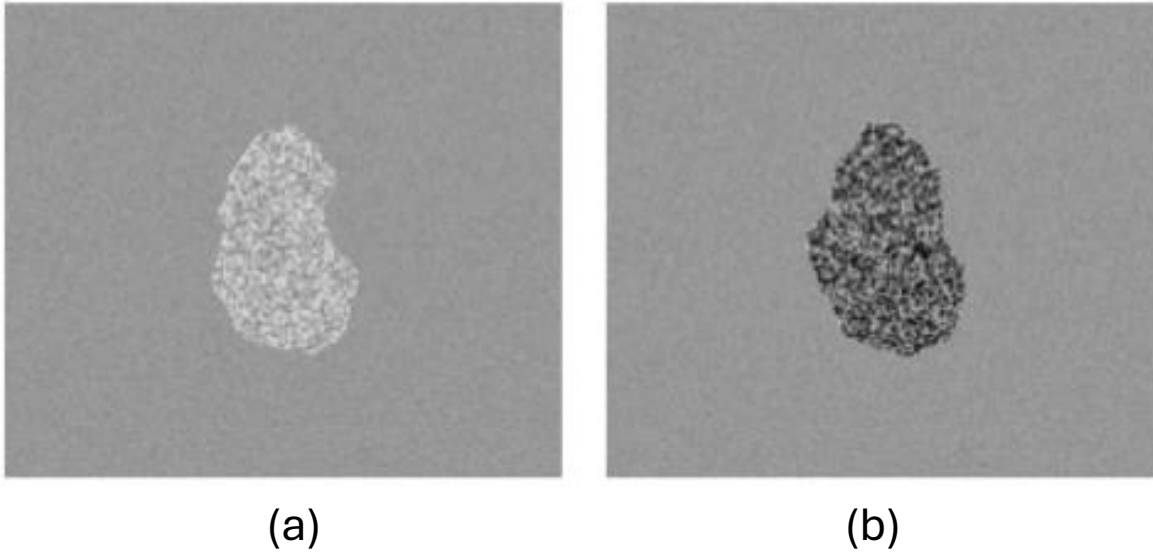
# Semantic Gap in Image Understanding



Grzegorz Marcin & Doniec Rafał, (2024). *Pattern Recognition*. University: Universität zu Lübeck



# Example for Medical Image Classification



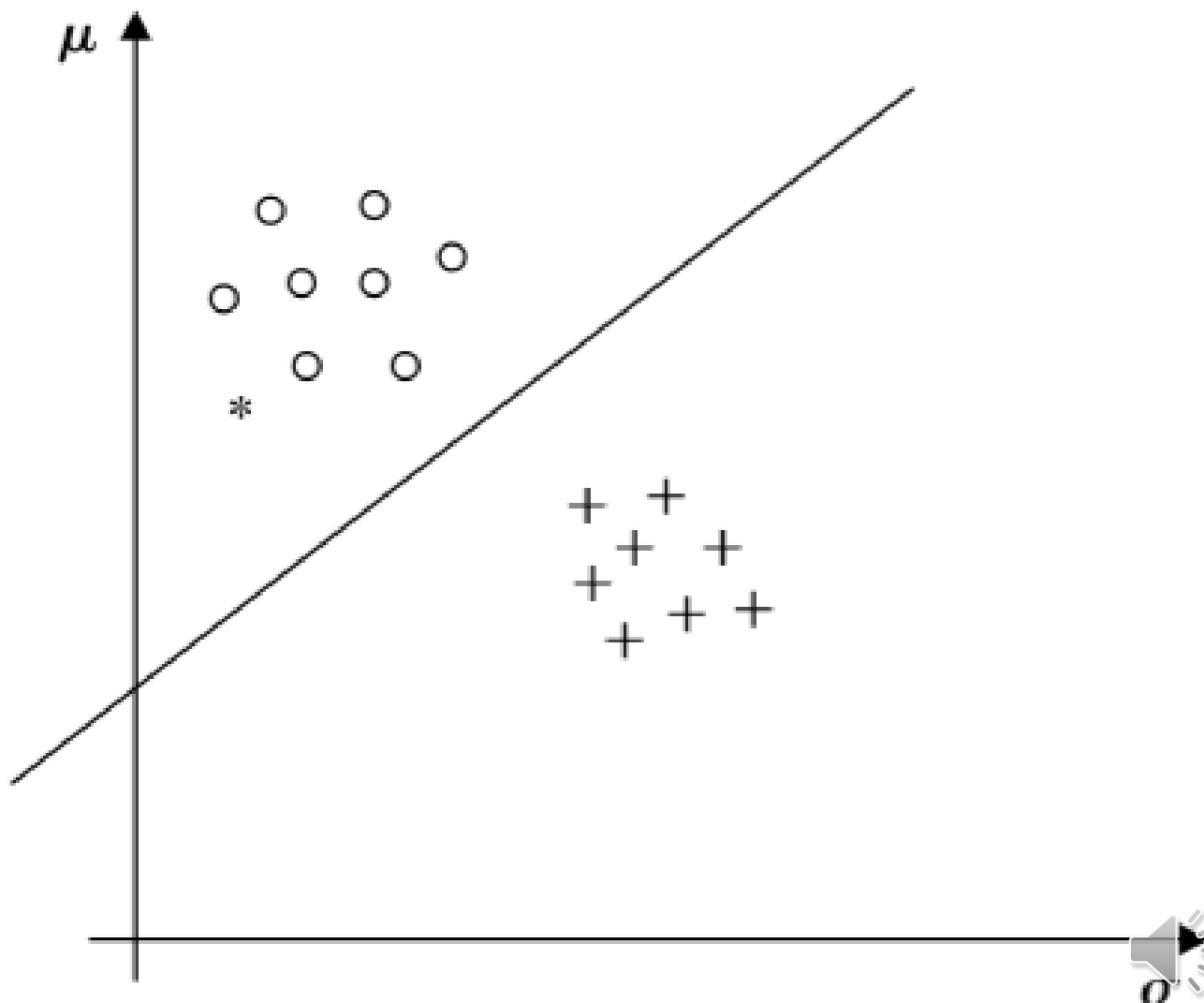
Examples of image regions corresponding to (a) class A and (b) class B.



# Example

## Descriptors for the Image Regions

- Plot of the mean value  $\mu$  and standard deviation  $\sigma$  for a number of different images originating from class A ( $\circ$ ) and class B (+).



# Feature Vectors → Random Vectors

Each feature vector identifies  
a single pattern (object)



Descriptors are called  
feature vectors

$$X = [x_1, x_2, \dots, x_l]^T$$



Feature vectors are treated  
as random vectors



# Signal Acquisition - Stochastic Process

- Stochastic processes are processes that proceed randomly in time.
- Rather than consider fixed random variables  $X$ ,  $Y$ , etc. or even sequences of i.i.d random variables, we consider sequences  $X_0, X_1, X_2, \dots$ . Where  $X_t$  represent some random quantity at time  $t$ .
- In general, the value  $X_t$  might depend on the quantity  $X_{t-1}$  at time  $t-1$ , or even the value  $X_s$  for other times  $s < t$ .



# Signal Acquisition - Stochastic Process

## Example



$$f(120, 180) = 219$$



$$f(120, 180) = 210$$



$$f(120, 180) = 208$$



$$f(120, 180) = 204$$



$$f(120, 180) = 198$$

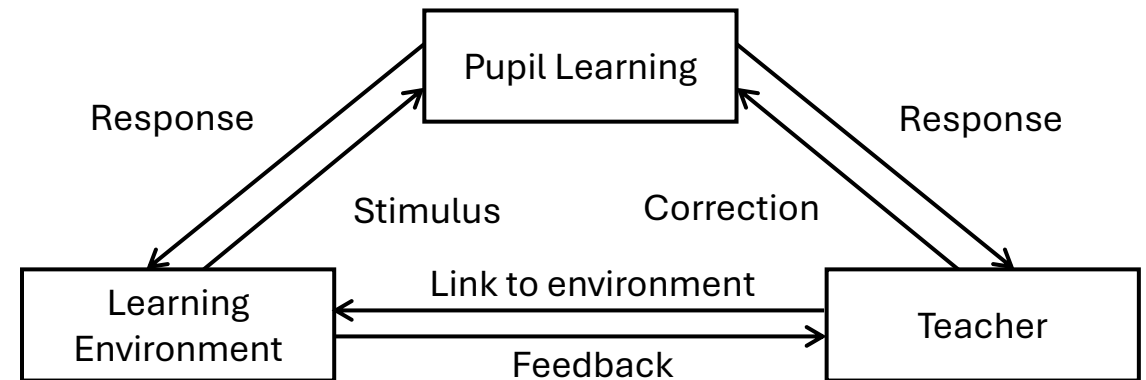




# Learning Strategies

## Supervised Learning

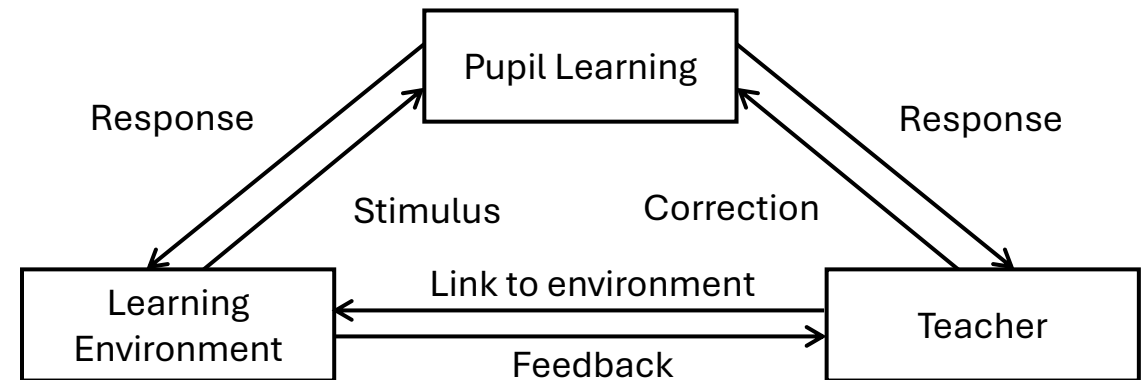
assumes that a set of labelled training data is available and the classifier is designed by exploiting this a-priori known information.



# Learning Strategies

## Supervised Learning

assumes that a set of labelled training data is available and the classifier is designed by exploiting this a-priori known information.



# Learning Strategies

## Semi-supervised Learning

applies both the labelled and unlabelled training for designing a classification system.



# Statistical Classification - Problem Statement

**Classification of an unknown pattern in the most probable of the classes!**

- Set of classes:  $\{\omega_1, \omega_2, \dots, \omega_M\}$
- Unknown pattern represented by its feature vector  $x$
- Conditional probabilities:  $P(\omega_i|x)$ ,  $i = 1, 2, \dots, M$
- Classification result: the class with the maximum conditional probability

**But how to compute the conditional probability for a particular class?**



# Probability P vs. Density p

## Probability P

is a real number describing an event belonging to the range  $<0,1>$ .

## Density p

is a value of a function<sup>1</sup>  $p(x)$  describing the distribution of the random variable  $x$ .

**If the random variable takes only discrete values, the densities become probabilities!**

<sup>1</sup>This function is often referred as pdf – probability density function.





# Bayes Decision Theory



# A Priori Probability vs. A Posteriori Probability

## A priori probability – probability before classification

- How probable is a particular class  $\omega_i$  for a pattern  $\mathbf{x}$  before applying any classification algorithm?
- Answer:  $P(\omega_i)$

## A posteriori probability – probability after classification

- How probable is a particular class  $\omega_i$  for a pattern  $\mathbf{x}$  after applying any classification algorithm?
- Answer:  $P(\omega_i|\mathbf{x})$



# Likelihood Density Function

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## Likelihood Density Function

How feature vector  $\mathbf{x}$  are distributed in a class  $\omega_i$ ?

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Answer:  $p(\mathbf{x}|\omega_i)$

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$p(\mathbf{x}|\omega_i)$  is the likelihood function of  $\omega_i$  with respect to  $\mathbf{x}$

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$p(\mathbf{x}|\omega_i)$  can be trained from examples

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# Two-class Problem

## Known

- Classes:  $\{\omega_1, \omega_2\}$
- A priori probabilities:  $P(\omega_1)$  and  $P(\omega_2)$
- Likelihood density functions:  $p(x|\omega_1)$  and  $p(x|\omega_2)$
- Pattern to be classified:  $x=[x_1, x_2, \dots, x_l]^T$

## Assumption

- The feature vectors can take any value in the  $l$ -dimensional feature space:  $x=[x_1, x_2, \dots, x_l]^T \in \mathbb{R}$

## Unknown

- A posteriori probabilities:  $P(\omega_1 | x)$  and  $P(\omega_2 | x)$



# Computation of the A Posteriori Probability

Using the Bayes Rule

$$P(\omega_i|x) = \frac{p(x|\omega_i)P(\omega_i)}{p(x)}, \quad i = 1, 2 \quad (1)$$

$p(x)$  – density function for  $x$





# Bayes Classification Rule (1)

Higher a posteriori probability wins

If  $P(\omega_1|x) > P(\omega_2|x)$ ,  $x$  is classified to  $\omega_1$

If  $P(\omega_1|x) < P(\omega_2|x)$ ,  $x$  is classified to  $\omega_2$



# Bayes Classification Rule (2)

Considering the Bayes Rule (Eq. 1)

If  $\frac{p(x|\omega_1)P(\omega_1)}{p(x)} > \frac{p(x|\omega_2)P(\omega_2)}{p(x)},$  x is classified to  $\omega_1$

If  $\frac{p(x|\omega_1)P(\omega_1)}{p(x)} < \frac{p(x|\omega_2)P(\omega_2)}{p(x)},$  x is classified to  $\omega_2$



# Bayes Classification Rule (3)

$p(x)$  can be discarded, because it is the same for all classes

If  $p(x|\omega_1)P(\omega_1) > p(x|\omega_2)P(\omega_2)$ ,  $x$  is classified to  $\omega_1$

If  $p(x|\omega_1)P(\omega_1) < p(x|\omega_2)P(\omega_2)$ ,  $x$  is classified to  $\omega_2$



# Bayes Classification Rule (4)

If the a priori probabilities are equal:  $P(\omega_1) = P(\omega_2)$

If  $p(x|\omega_1) > p(x|\omega_2)$  ,  $x$  is classified to  $\omega_1$

If  $p(x|\omega_1) < p(x|\omega_2)$  ,  $x$  is classified to  $\omega_2$

We are done, since the likelihood density functions  $p(x|\omega_1)$  and  $p(x|\omega_2)$  are assumed to have been trained from examples!

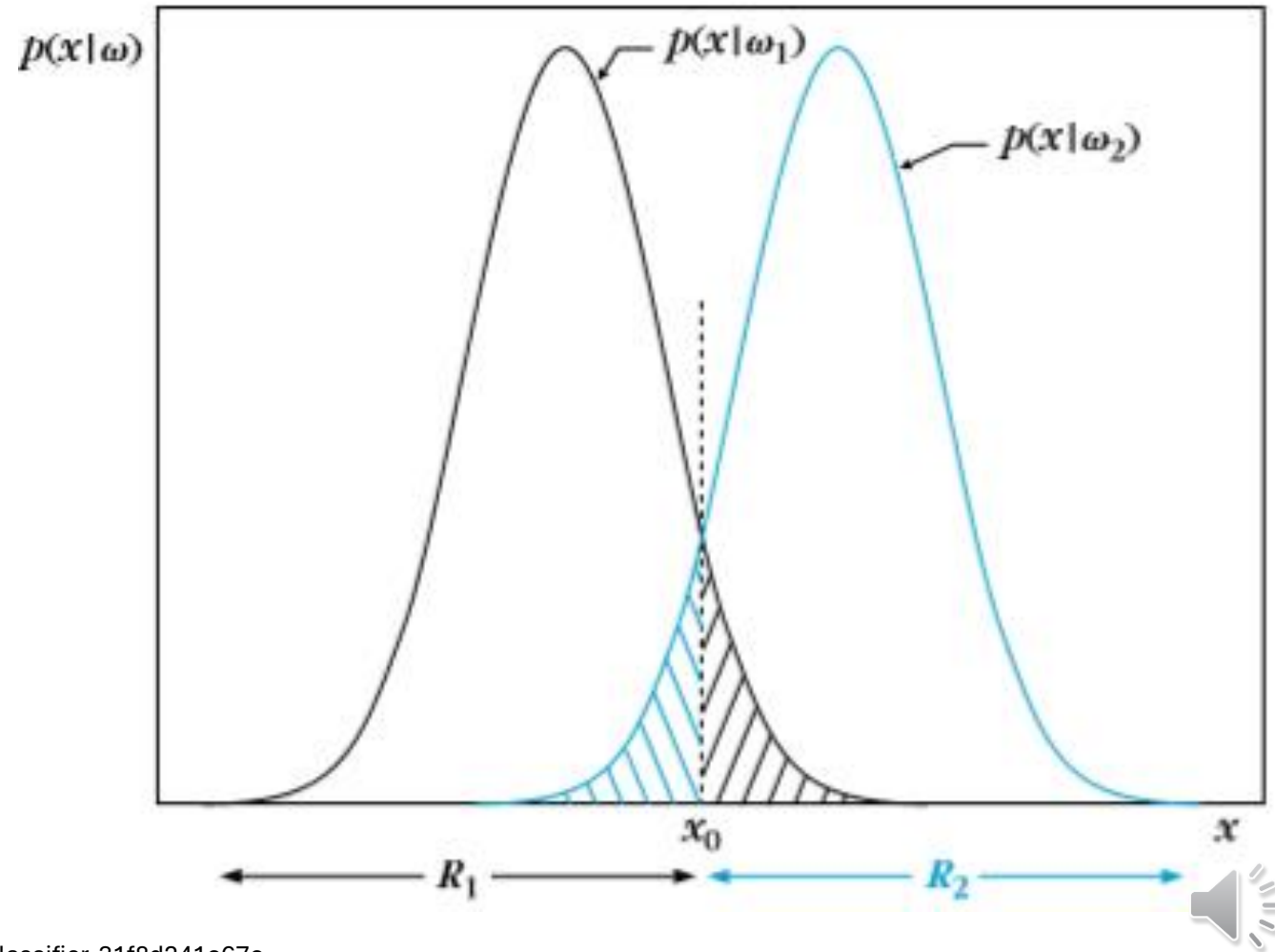


# Classification Error Probability

Error Probability:

$$\frac{1}{2} \int_{-\infty}^{x_0} p(x|\omega_2) dx + \frac{1}{2} \int_{x_0}^{\infty} p(x|\omega_1) dx$$

$P_e =$



Source: <https://medium.com/@thommaskevin/tinyml-gaussian-naive-bayes-classifier-31f8d241c67c>

# Classification Error Probability in General

- A priori probabilities are not equal:  $P(\omega_1) \neq P(\omega_2)$
- Feature vectors have more than one dimension:  $l > 1$

$$\mathbf{x} = [x_1, x_2, \dots, x_l]^T$$

- General form:

$$P_e = P(\omega_1) \int_{R_2} p(\mathbf{x}|\omega_1) d\mathbf{x} + P(\omega_2) \int_{R_1} p(\mathbf{x}|\omega_2) d\mathbf{x}$$





# Pattern Recognition Quiz

## 1. What is Pattern Recognition?

- A. Pattern recognition is identifying patterns without using any pre-learned information.
- B. Pattern recognition is the classification of data based on previously gained knowledge or statistical information extracted from patterns.

## 2. Is Speech Recognition an example of Pattern Recognition?

- A. Yes, as it involves processing raw data and classifying patterns for machine use.
- B. No, speech recognition doesn't involve identifying or classifying patterns.

## 3. What is the Difference Between Classification and Clustering?

- A. Classification assigns labels based on training patterns, while clustering groups data without predefined labels.
- B. Classification and clustering are the same process, as both involve labeled data.

## 4. Can Binary Quantities be Used as Features?

- A. No, features can only be represented as continuous variables.
- B. Yes, features can be continuous, discrete, or binary variables.

## 5. How are Features Obtained?

- A. Features are randomly generated without any measurement criteria.
- B. A feature is a function of measurements that quantify significant characteristics of an object.



# Pattern Recognition Quiz with Correct Answers

## 1. What is Pattern Recognition?

- A. Pattern recognition is identifying patterns without using any pre-learned information.
- B. Pattern recognition is the classification of data based on previously gained knowledge or statistical information extracted from patterns.

*Answer: B*

## 2. Is Speech Recognition an example of Pattern Recognition?

- A. Yes, as it involves processing raw data and classifying patterns for machine use.
- B. No, speech recognition doesn't involve identifying or classifying patterns.

*Answer: A*

## 3. What is the Difference Between Classification and Clustering?

- A. Classification assigns labels based on training patterns, while clustering groups data without predefined labels.
- B. Classification and clustering are the same process, as both involve labeled data.

*Answer: A*

## 4. Can Binary Quantities be Used as Features?

- A. No, features can only be represented as continuous variables.
- B. Yes, features can be continuous, discrete, or binary variables.

*Answer: B*

## 5. How are Features Obtained?

- A. Features are randomly generated without any measurement criteria.
- B. A feature is a function of measurements that quantify significant characteristics of an object.

*Answer: B*

Thank you for your  
attention

